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INSTRUCTIONAL OBJECTIVES FOR A JUNIOR COLLEGE COURSE IN  
COLLEGE ALGEBRA

ED 033683

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**COLLEGE ALGEBRA OBJECTIVES: SET # 1**

## UNIT I

### Review of Algebra and Mathematical Logic

Certain background in high school algebra is assumed in this course. Fundamentals such as factoring the difference of two squares will be used frequently without additional explanation. A review of these basic techniques will facilitate understanding later in the course.

Schedule: pretest; fractions, signed numbers, terminology (1 hr.)  
factoring (1 hr.)  
methods of proof and mathematical reasoning (1 hr.)

**I. GOAL:** The student will understand the basic symbols and terminology of algebra.

**OBJECTIVE:** 1. Given a list of common mathematical symbols <sup>he</sup> and will select the correct definition or explanation of each. (90%)

2. Given a list of theorems, he will identify the hypothesis and conclusion of each. (90%)

3. Given a list of mathematical statements similar to ones in the preface of the text, he will select the converse to each from a choice of three possible converses. (90%)

**II. GOAL:** The student will know simple factoring methods.

**OBJECTIVE:** 1. Given a list of polynomials, he will select the best method of factoring each (removal of a common factor, perfect square, difference of squares, difference or sum of cubes) (80%)

2. Given a list of algebraic expressions, he will rewrite them in standard form. (90%)

3. Given a list of polynomials, not necessarily in standard form, he will factor them into irreducible factors. (70%)

**III. GOAL:** The student will know different methods of proofs.

**OBJECTIVE:** 1. Given a list of short proofs, he will correctly identify the method of proof (direct, contradiction, construction, contrapositive) (80%)

2. Outside of class he will read one article from the list below. He shall then hand in a 25-100 word summary of major points discussed in the article he selected. (100%)

## UNIT II

Set Theory

Set theory underlies all mathematical thought. The idea of restricting one's attention to a group of elements with some common property is the idea of a set. The student already has an intuitive concept of "set." Now he will learn elements of the formal study of sets. The terminology and notation will be explained. The operations of union, intersection, and complementation will be studied in an attempt to encourage precision and logical analysis in mathematics.

Schedule: concept of set; description and notation (1 hr.)  
 operations with sets (1 hr.)  
 set identities (1 hr.)

I. GOAL: The student will know the properties of union, intersection, and complementation of sets.

OBJECTIVE: 1. Given verbal description of sets, he will rewrite the sets in "set notation." (90%)

2. Given a listing of elements in three sets, he will write the elements in various combinations of the three sets, such as  $A \cap (B \cup C)$  (90%)

3. He will match common symbols from set theory with their definitions. (e.g.  $A \cap B$  -  $x \in A \text{ and } x \in B$ ) (90%)

II. GOAL: The student will understand the use of Venn diagrams.

OBJECTIVE: 1. Given an expression such as  $A \cap B$  he will select the correct Venn diagram to represent it. (80%)

2. Given a set identity, he will construct a Venn diagram for each side of the identity and compare the results. (100%)

III. GOAL: The student will be able to prove set theoretic identities.

OBJECTIVE: 1. Outside of class, he will prove the following:

$$A \cup \emptyset = A$$

$$A \cap \emptyset = \emptyset$$

$$A - B = A \cap B'$$

$$A \cap A = A$$

$$A \cap A' = \emptyset$$

$$A \cup \emptyset = A$$

$$A \cap B = B \cap A$$

$$A \cup A' = U$$

$$A \cap (B \cup C) = A \cap (B \cup C)$$

(70%)

## UNIT III

### The Number System

Algebra uses mathematical symbols to represent numbers. Thus, all of algebra is based in the properties of the numbers themselves. Mostly, one restricts study to the real numbers, that is, numbers which can be the square roots of positive numbers. There are many properties of real numbers that lead to this characterization.

Additional study of quadratics reveals that some algebraic equations have no real solutions.  $x^2 = -1$  is one famous example. The complex numbers were invented to represent the roots of such equations. Every algebraic equation has at least one complex root (reals are considered special cases of complex numbers). Thus, the study of complex numbers is important in college algebra.

<u>Schedule:</u>	integers; numerology; number theory	(1 hr.)
	basis systems	(1 hr.)
	rationals and reals; zero	(1 hr.)
	powers and roots; $\sqrt{2}$ is irrational	(1 hr.)
	complex arithmetic	(1 hr.)
	Cartesian plane and complex numbers	(1 hr.)

I. GOAL: The student will know the various types of numbers and and their relationship to one another.

OBJECTIVE: 1. Given a list of numbers, he will identify them as natural numbers, integers, rational, reals or complex. (90%)

2. He will construct an addition or multiplication table for one of the bases two through twelve. (80%)

II. GOAL: The student will understand what is meant by prime factorization.

OBJECTIVE: 1. Given a list of numbers between 2 and 400, he will identify them as prime or composite. (90%)

2. Given a list of numbers between 2 and 400, he will write them in their prime factorization. (60%)

## UNIT IV

### Functions, Relations, and Graphs

A function is a special kind of set. It consists of ordered pairs of numbers  $(x, y)$  in which the second number,  $y$ , is related to the first by some rule. For example,  $y = 2x + 1$  is a function. The concept of function is perhaps second only to the concept of set in importance.

A function is a special kind of relation. Thus, one can establish a mathematical study of relations as well as functions. One might wonder, for example, if one number is related to a second, how is the second related to the first. Is it related in the same way? Since functions are relations, the properties of relations will carry over and apply to functions.

One way to study functions and relations is to try to visualize them through graphs. A graph is a picture of the function, in a sense. The method can be extremely useful in finding out broad properties of the function or relation, such as its maximum, how much it wiggles, and so on.

<u>Schedule:</u> notation; application in science; rectangular coordinates.	(1 hr.)
graphs of equations and functions	(1 hr.)
maps; inverses	(1 hr.)
relations	(1 hr.)
inequalities	(2 hr.)
continuous functions	(1 hr.)
graphical solution of equations	(1 hr.)
special functions	(1 hr.)

I. GOAL: The student will be able to graph ordered pairs.

OBJECTIVE: 1. Given a list of ordered pairs, the student will indicate the correct point corresponding to each pair on a Cartesian plane. (90%)

2. Given several points on a Cartesian plane, he will write the ordered pair for each point. (90%)

II. GOAL: The student will understand graphs of functions.

III. GOAL: The student will understand the order relationship of the real numbers.

OBJECTIVE: 1. Given sets of real numbers, he will place the members of each in correct order from smallest to largest. (90%)

2. Outside of class, he will look up the trichotomy principle and explain in 25 to 200 words how it relates to the order relationship. (100%)

IV. GOAL: The student will know how to add, subtract, divide, and multiply complex numbers. He will know the Cartesian and polar forms for complex numbers.

OBJECTIVE: 1. The student shall complete the exercises in the program "Complex Numbers" and turn it in. Work to be done outside of class and handed in at the beginning of meeting 5. (100%)

OBJECTIVE: Given examples of functions, he will find the domain and range of each function. (80%)

2. He will select the correct graph from several for a given function. (100%)

3. He will read appropriate values of a function at selected points, given the graph of the function. (90%)

III. GOAL: The student will know how to find the inverse of a function.

OBJECTIVE: 1. Given a list of functions, he will select the proper inverse function of each. (70%)

IV. GOAL: The student will know how to solve inequalities.

OBJECTIVE: 1. Given a list of operations with inequalities, he will select the theorem that justifies the particular operation. (example:  $\frac{a}{b} > \frac{c}{d} \rightarrow ad > bc, ab > 0 \Rightarrow \frac{1}{a} < \frac{1}{b}$ ) (60%)

2. Given a list of simple inequalities, he will select the correct solution set for each. (50%)

3. Given two or three inequalities, he will find the graphical solution for each. (50%)

V. GOAL: The student will have an intuitive concept of continuity.

OBJECTIVE: 1. Given graphs of several functions, he will identify them as continuous or discontinuous. (90%)

VI. GOAL: The student will solve equations by graphical methods.

OBJECTIVE: 1. Given a graph of an equation, he will locate zeroes, local maxima and local minima. (60%)

VII. GOAL: The student will know properties of the absolute value, greatest integer, and constant function.

OBJECTIVE: 1. The student will construct a graph for examples of these three kinds of functions. (60%)

## UNIT V

### Quadratics

The theory of quadratic or second-degree equations and functions is more complete than that for higher order functions. Much of this theory is treated in high school algebra; some new material will be added now. Many problems in engineering, business, and other fields can be reduced to the solving of a quadratics equation. Moreover, studying the nature of the roots and the methods of solution for quadratics will give us some model to turn to in the study of higher order equations. A thorough understanding of quadratic equations is necessary for these reasons.

<u>Schedule:</u> factoring; completing the square; quadratic formula	(1 hr.)
nature of roots; simultaneous equations	(2 hr.)
reducing word problems to quadratics	(1 hr.)

I. GOAL: The student will be able to solve any quadratic equation.

OBJECTIVE: 1. Given a list of quadratics, the student will find all roots; real and imaginary. (80%)

2. He will write the quadratic formula. (100%)

II. GOAL: The student will be able to set up problems into the form for a quadratics equation.

1. Given verbal problems like those in the text, he will set them up for solution; he need not actually solve them. (50%)

2. Given fractional equations or irrational equations, he will reduce them to quadratic form. (70%)

III. GOAL: The student will be able to solve simultaneous equations in two unknowns; equations linear or quadratic.

OBJECTIVE: 1. Given sets of two equations in two unknowns, one quadratic, the other linear or quadratic, he will solve for both unknowns in each set. (60%)

## UNIT VI

### Polynomials and Equations.

Polynomial equations are one of the subjects of college algebra. Quadratic equations are a special case of these - second degree polynomial equations. The general polynomial is usually written as  $a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$ . The questions one asks about polynomials are the same as the ones for quadratics. How many roots are there? Are they real or imaginary? What is the graph of the function?

Unfortunately, many of these questions cannot be answered for higher order polynomial equations. Part of the problem is that factorization is not always obvious; There are ways to check for certain factorizations, however. Synthetic division is one such method. There are also ways of approximating the roots, finding how many are positive or negative, and other such general information.

<u>Schedule:</u>	elementary operation with polynomials; fundamental theorem	(1 hr.)
	Remainder and Factor Theorems	(1 hr.)
	synthetic division	(1 hr.)
	number of roots; conjugate complex roots	(1 hr.)
	nature of roots; rational graphic solutions	(1 hr.)
	upper and lower bounds; reversing sign	(1 hr.)
	Descartes' Rule of Signs	(1 hr.)
	Successive approximations	(2 hrs.)
	Roots and Coefficients	(1 hr.)
	Review	(1 hr.)

I. GOAL: The student will understand rational expression with polynomials.

OBJECTIVE: 1. Given two polynomials, he will form their sum, difference, product or quotient. (80%)

II. GOAL: The student will be able to find real roots of a polynomial.

OBJECTIVE: 1. Given a list of polynomials together with a monomial expression, he will determine whether the monomial is a factor of the polynomial. (90%)

2. Given a partial list of roots for a polynomial equation, he will state how many real roots are missing still from the list. (60%)

3. Given a polynomial equation, he will write it as the product of irreducible factors. (100%)

**III. GOAL:** The student will use graphical methods of solution to determine roots of polynomials.

**OBJECTIVE:** 1. Given a polynomial and its graph, he will determine one real root to two decimal places. (90%)

2. Given the graph of a polynomial, he will state how many real roots the polynomial has. (90%)

**IV. GOAL:** The student will fix upper and lower bounds on the roots of equations, if possible to do this.

**OBJECTIVE:** 1. Given a list of polynomials, he will find the upper and lower bounds for the roots of each. (70%)

**V. GOAL:** The student will know Descartes' Rule of Signs.

**OBJECTIVE:** 1. Given a list of polynomials, he will indicate the number of positive and negative roots. (90%)

2. He will write a proof that  $\sqrt{2}$  is not rational using Descartes' Rule of Signs as a basis. (100%)

**VI. GOAL:** Given any polynomial with rational coefficients, the student will be able to solve it.

**OBJECTIVE:** 1. Given a list of polynomials, he will state by what method each polynomial's roots are best found (factor theorem, examining coefficients, reversal of sign, successive approximation) (40%)

2. Given a polynomial, he will find one of its roots to six decimal places. (90%)

## UNIT VII

### Determinants and Systems of Linear Equations

Frequently, one will encounter problems having three or four unknowns. If the unknowns are all in first degree expressions, such systems can be solved. A system of such equations considered together is called a system of simultaneous linear equations. Determinants provide a quick method for solving such systems.

#### I. The student will use determinants to solve linear equations.

OBJECTIVE: 1. Given a system of three simultaneous linear equations in three unknowns, he will solve for all three unknowns using determinants. (100%)

2. Given a system of four linear equations in up to four unknowns, he shall state whether the system is consistent or inconsistent, whether it is dependent or independent and if dependent, how many free unknowns there are. (90%)

#### II. GOAL: The student will know how to use determinants in fields other than algebra.

OBJECTIVE: 1. Given the coordinates of the vertices of a list of triangles, he will use determinants to find the areas of the triangles. (70%)

2. The student will construct an example of a business problem that could be solved by linear programming methods; to be done outside of class and the various examples will be discussed in class. (100%)

#### III. GOAL: The student will know something of the theory of determinants.

OBJECTIVES: 1. Given a list of terms such as rank, column, and so on, the student will select the best definition for each term. (80%)

2. Given a list of statements about determinants, the student will indicate whether the statement is true or false. (90%)

3. Given a list of properties of determinants, outside of class he will construct an example to illustrate each property. (70%)

COLLEGE ALGEBRA OBJECTIVES: SET # 2

## Materials of Instruction

### **Required Text:**

Fisher, Robert C. and Allen D. Ziebur, Integrated Algebra and Trigonometry with Analytic Geometry, 2nd ediction, Prentice Hall, 1967.

### **Recommended References:**

Allendoerfer and Oakley, Fundamentals of Freshman Mathematics, McGraw-Hill, 1965.

Beckenbach and Drooyan, Modern College Algebra and Trigonometry, Wadsworth, 1968.

Swokowski, Earl, Algebra and Trigonometry, Prindle, Weber and Schmidt, 1967.

The following outlines may also be of use:

College Outline Series, Barnes and Noble

Plane and Spherical Trigonometry

\*Modern Trigonometry

Littlefield, Adams Series

College Algebra

Analytic Geometry

Shaum Outline Series

\*First Year College Mathematics

\*College Algebra

\*Highly recommended

## MAJOR CONCEPTS TO BE LEARNED

### I. Review of the Real Number System

In this unit the student will learn about number systems, inequalities, exponents, and the concept of sets.

### II. Functions

The student will learn fundamentals of graphing, proportion, and just what a function is.

### III. Exponents and Logarithms

The student will be introduced to logarithmic and exponential functions, the use of tables and interpolation.

### IV. Trigonometric Functions

The student will learn the elements of trigonometry from the right-angle and numeric points of view.

He will be introduced to geometric interpolations of trigonometric functions.

### V. Complex Numbers

The student will be introduced to the arithmetic of a two-dimensional extension of the real numbers.

### VI. Equations

The student will learn about polynomials and their roots.

## **VII. Systems of Equations**

The student will learn about elementary properties of matrices and determinants. He will be introduced to the solution of a system of equations.

## **VIII. Combinations and Permutations**

The student will learn about combinations, the factorial and sigma notations and the binomial theorem.

## **IX. Sequences**

The student will learn what a sequence is. Also, he will learn about arithmetic and geometric progressions and means.

## **X. Inverse Functions and Trigonometric Equations**

The student will be introduced to arc-functions and the solution of trigonometric equations.

## **XI. Analytic Geometry**

The student will learn how to translate and rotate coordinate axes. He will become familiar with various conic sections and be introduced to polar coordinates.

## THE SYSTEM OF REAL NUMBERS

GOAL: The student will understand associativity, distributivity, and commutativity.

Objective: He will solve a set of problems such as those given on p. 4-5 of text as a homework assignment (80%).

GOAL: The student will understand division and cancellation.

Objective: He will solve problems such as those given on p. 8-9 as homework (80%).

GOAL: The student will understand the concept of and need for irrational numbers.

Objective: He will answer, as homework, questions such as those found on p. 13-14 of the text (80%).

Objective: He will write a paper (100-200 words) outside of class in which he defines irrational numbers and gives at least three examples of irrational numbers (100%).

GOAL: The student will understand the basic concepts of set theory.

Objective: Given descriptions of physical situations, he will translate them into set notation (outside of

class, 90%).

Example: John is in Math 1, a class at Orange Coast college. Answer: John  $\in \{ \text{Math 1} \} \subseteq \{ \text{Orange Coast College} \}$ .

Objective: Given problems like those on p. 21-22, he will answer them outside of class (80%).

GOAL: The student will understand basic principles of inequalities (equation 4.1-4.5).

Objective: Outside of class, he will prove 4.3, 4.4, 4.5 (66%).

GOAL: The student will be able to solve simple inequalities.

Objective: He will write the solution sets to several inequalities outside of class (80%).

GOAL: The student will understand the basic facts about absolute value.

Objective: He will solve problems such as those on p. 29 of the text, outside of class (80%).

GOAL: The student will be able to eliminate absolute value signs from an inequality or to insert them.

Objective: The student will express inequalities containing absolute value signs without them and vice versa, outside of class (80%).

GOAL: The student will understand the laws of exponentiation for integer exponents.

Objective: The student will answer p. 36, #6, 7, 8, 9 outside of class (75%).

GOAL: The student will understand the generalized concept of exponents.

Objective: He will solve problems such as those on p. 39-40 outside of class (75%).

GOAL: The student will retain the knowledge gained in the study of Chapter 1.

Objective: At the end of the chapter he will answer questions on a quiz (closed book) made up of problems similar to the Review and Miscellaneous Problems in the chapter (80%).

## II

### FUNCTIONS

GOAL: The student will understand the concept of functions and the vocabulary of functions.

Objective: Given a list of functions, the student will write domain and range of each. Homework (80%).

Objective: In class (closed book), he will define "function" and give at least three examples stating domain and range of each (100%).

Objective: Given a set of problems like those on p. 47-48, 50-51, he will show them for homework (80%).

GOAL: The student will understand the concepts of a graph and a Cartesian coordinate system.

Objective: As homework, he will complete problems similar to those on p. 57 (80%).

GOAL: The student will be able to relate a function to a graph and vice versa.

Objective: Given a set of graphs, the student will write the functions to which they correspond (70%).

Objective: Given a set of functions, the student will graph them (80%).

GOAL: The student will understand linear functions and linear equations.

Objective: He will complete 10 problems similar to those on p. 69-70 (80%).

GOAL: The student will understand the concept of direct proportion.

Objective: As homework, the student will answer questions such as those on p. 64-65 (80%).

GOAL: The student will understand the concept of inverse proportion.

Objective: Given a list of functions, the student will state whether they are direct or inverse proportions and he will graph them (80%).

GOAL: The student will retain the knowledge gained in the study of Chapter 2.

Objective: At the end of the chapter, he will answer questions on a quiz (closed book) made up of problems similar to the Review and Miscellaneous Problems in the chapter (80%).

### III

#### EXPONENTIAL AND LOGARITHMIC FUNCTIONS

GOAL: The student will understand the concept of exponential functions and theorem B.1 of the text.

Objective: He will complete assigned problems from p. 80-81 (80%).

Objective: Given pairs of exponential functions, he will combine them as per theorem 18.1 (80%).

GOAL: The student will understand the concept of logarithm and the relationship between logs and exponential functions.

Objective: Given pairs of exponentials, he will write the corresponding log functions (80%).

Objective: Given a set of log functions, he will write the corresponding exponential (80%).

GOAL: The student will understand the fundamental properties of logs as exemplified in theorem 20.1, 20.2, 20.3, 20.4.

Objective: The student will solve problems like those on p. 86-87 (85%).

GOAL: The student will understand the relationship between logs of different bases and between a log

function and its corresponding exponential.

Objective: He will solve problems of the type on p. 90-91 (80%).

GOAL: The student will be able to use a log table.

Objective: Given a list of logs, he will find their antilogs and vice versa (80%).

GOAL: The student will be able to interpolate using log tables.

Objective: Given a set of logs, he will write the antilog to one significant figure beyond that provided in the table (80%).

GOAL: The student will understand logarithms as an aid to arithmetic calculation.

Objective: The student will evaluate a list of logs given appropriate tables, at home (90%).

Objective: He will evaluate antilogs with tables, at home (90%).

Objective: Given arithmetic problems, he will solve them using logs outside of class (90%).

GOAL: The student will understand how to solve equations whose unknowns are exponents or antilogs.

Objective: Given a set of equations like those

on p. 103-104, the student will solve for the unknown, outside of class (80%).

GOAL: The student will retain the knowledge gained in the study of Chapter 3.

Objective: At the end of the chapter, he will answer questions on a quiz (closed book) made up of problems similar to the Review and Miscellaneous Problems in the chapter (80%).

THE TRIGONOMETRIC FUNCTIONS

GOAL: The student will understand the geometry of the unit circle and the trigonometric point.

Objective: He will answer questions like those on p. 111-112 of text as homework (75%).

GOAL: The student will understand the trigonometric functions and the basic relations between them.

Objective: Given a list of trig identities like those in section 27, the student will prove them (75%).

GOAL: The student will be able to evaluate trig functions by use of trig functions.

Objective: He will answer questions from P. 118-119 as homework. (85%).

GOAL: The student will understand the symmetry of trigonometric functions.

Objective: Given a list of trig functions such as  $\sin(3.7)$ , the student will evaluate them as homework (85%).

GOAL: The student will understand the periodicity of the trigonometric functions.

Objective: He will answer questions from those on p. 128-129 as homework (80%).

GOAL: The student will understand the trigonometric addition formulas.

Objective: Given a list of addition formulas, he will prove them as homework (70%).

GOAL: The student will understand the multiple angle formulas.

Objective: He will complete problems from p. 138-139 as homework (80%).

GOAL: The student will have an understanding of trigonometric identities and know how to derive them.

Objective: He will prove a set of trig identities as a homework assignment (70%).

GOAL: The student will demonstrate an understanding of elementary trigonometry.

Objective: Under test conditions, he will prove or derive trig identities, graph trig functions, and evaluate trig functions and work applications such as those in problem sets 26-34 (75%).

GOAL: The student will understand the concept of the general sine wave.

Objective: He will define, in writing, in class, the following: period, frequency, phase shift, amplitude (100%).

Objective: Given the values of period, frequency, phase shift, and amplitude, he will write the appropriate constants, A, a, b from  $y = A \sin(ax + b)$ .  
Homework, (90%).

GOAL: The student will understand the relationship between radians and degrees.

Objective: Given a set of degrees, the student will write the appropriate radians and vice versa at home (80%).

GOAL: The student will understand the concept of arc length on a circle.

Objective: Given central angle and radius, he will determine arc length, at home (100%).

Objective: He will work problems from among those on p. 155-156 at home (80%).

GOAL: The student will understand the geometric foundations of trigonometry.

Objective: The student will solve problems at home from p. 161-162 (80%).

GOAL: The student will understand the concepts of right-angle trigonometry.

Objective: Graphically, he will interpret the trig functions of a right-angle onto the unit circle at home (85%).

Objective: Given the length of sides of triangles, he will evaluate the trig functions in class (90%).

GOAL: The student will understand the Law of Sines and its usefulness.

Objective: He will prove the Law of Sines for a right triangle, at home (100%).

Objective: He will solve applications problems from (or similar to) those on p. 169-170 at home (70%).

GOAL: The student will understand the Law of Cosines and its usefulness.

Objective: He will state the specific corollary to the law which applies to right angles, in writing in class (100%).

Objective: He will solve applications problems from (or similar to) those on p. 172-173 at home (70%).

GOAL: The student will be able to determine the area of a polygonal surface.

Objective: Given the angles and sides of a polygonal surface, the student will find the area of that surface and write it as homework (85%).

GOAL: The student will retain the knowledge gained in the study of chapter 4.

Objective: At the end of the chapter, he will answer questions on a quiz (closed book) made up of problems similar to the Review and Miscellaneous Problems in the chapter (80%).

## THE COMPLEX NUMBERS

GOAL: The student will understand the arithmetic of complex numbers.

Objective: He will solve addition and multiplication problems in the complex field at home (90%).

Objective: He will identify the parts of a complex number in class, in writing (100%).

GOAL: The student will understand the concept of complex conjugate.

Objective: He will solve complex division problems at home (80%).

Objective: He will prove:

$$(1) \overline{uv} = \overline{u} \overline{v}, \quad \overline{u+v} = \overline{u} + \overline{v}$$

$$(2) z = \overline{z} \Leftrightarrow z \text{ is real}$$

GOAL: The student will be able to relate complex numbers to the plane.

Objective: Given a set of complex numbers in the form of 2-tuples, he will write them in trigonometric form and vice versa, at home (90%).

Objective: Given a set of complex numbers he will plot them at home (100%).

GOAL: The student will be able to find the roots of a complex number.

Objective: The student will write the n-th roots of given numbers (n specified) and graph all roots (80%).

GOAL: The student will retain the knowledge gained in the study of chapter 5.

Objective: At the end of the chapter he will answer questions on a quiz (closed book) made up of problems similar to the Review and Miscellaneous Problems in the chapter (80%).

VI.

THEORY OF EQUATIONS

GOAL: The student will understand the basic concepts and arithmetic operations for polynomials.

Objective: Given a set of arithmetic problems involving polynomials, he will solve them, at home (90%).

GOAL: The student will be able to solve linear equations.

Objective: He will solve the linear equations on a list provided as homework (90%).

GOAL: The student will understand the basic concepts and methods of solutions of quadratic polynomials.

Objective: Given a list of polynomials, he will find their roots by the method specified (complete the square, factoring, quadratic formula) at home (90%).

Objective: Without calculating the roots, he will determine whether they are real, complex, multiple, at home (80%).

GOAL: The student will understand the concepts of quadratic solution of polynomials as applied to other-order polynomials.

Objective: Given a list of polynomials, he will solve them by quadratic means, at home (80%).

Objective: Given a set of multinomials (like polynomials but with fractional exponents), he will solve them by quadratic means, at home (80%).

GOAL: The student will understand the factoring method of solving quadratic polynomials.

Objective: Given a set of polynomials, he will write them in factored form, at home (80%).

GOAL: The student will understand the concepts of division and factorization in polynomials of degree higher than two.

Objective: Given a set of polynomials  $P(x)$  and  $r$ , he will write  $Q(x)$  and  $R$  such that  $P(x) = Q(x)(x-r) + R$ , at home (90%).

Objective: He will write the answers to a set of polynomial divisions, at home (90%).

Objective: He will use the Factor Theorem to write proofs that given quantities are factors of given polynomials, at home (80%).

GOAL: The student will know the characteristics of roots of polynomials.

Objective: Given a set of polynomials, he will characterize their roots, at home (80%).

GOAL: The student will know the characteristics of roots (or factors) of polynomials with real coefficients.

Objective: Given a subset of the roots of polynomials, he will write the polynomial with real coefficients of lowest degree which has these as some of its roots, at home (80%).

Objective: He will prove: Every cubic polynomial with real coefficients has at least one real root, at home (100%).

Objective: Given a list of polynomials, he will write them as the product of the 1st and 2nd degree polynomials with real coefficients, at home (80%).

Objective: Given a list of polynomials, he will find all roots, at home (85%).

GOAL: The student will know the characteristics of roots of polynomials with integral coefficients.

Objective: He will find all rational roots of given polynomials, at home (85%).

GOAL: The student will be able to determine the regions in which a root to a real polynomial exists.

Objective: Given a list of polynomials, he will write the integers between which roots exist, at home (75%).

Objective: Given a list of polynomials, he will write all roots, correct to 1 decimal place, at home (75%).

GOAL: The student will retain the knowledge gained in the study of chapter 6.

Objective: At the end of the chapter, he will answer questions on a quiz (closed book) made up of problems similar to the Review and Miscellaneous Problems in the chapter (80%).

VII

SYSTEMS OF EQUATIONS

GOAL: The student will be able to transform a set of equations into an equivalent set of equations.

Objective: The student will write whether or not given number pairs are solutions to given systems, at home (80%).

Objective: The student will solve systems of equations and write the solutions, at home (75%).

Objective: The student will write whether or not pairs of systems of equations are equivalent, at home (75%).

GOAL: The student will understand triangular form as a means of solving systems of equations.

Objective: Given systems of equations, he will write them in equivalent triangular form and solve them, at home (80%).

GOAL: The student will understand the correspondence between the system of equations and the matrix.

Objective: The student will solve systems of equations, at home, by matrix means (85%).

GOAL: The student will understand the types of systems of equations (see Section 60).

Objective: As a quiz, he will define determinative, dependent, inconsistent, and identify to which category given graphs and systems belong (90%).

Objective: As homework, given a set of systems, he will write their type and graph them (80%).

GOAL: The student will be able to solve systems involving higher order polynomials.

Objective: Given systems including quadratic equations, the student will solve them as homework (85%).

GOAL: The student will know the concept of a determinant.

Objective: Given a set of matrices, he will evaluate their determinants, at home (85%).

Objective: Given a set of systems, he will evaluate their determinants, at home (85%).

Objective: Given sets of equivalent systems, he will evaluate each of their determinants, at home (90%).

Objective: Given a set of determinants, he will evaluate them, at home (90%).

GOAL: The student will be able to evaluate determinants of any order.

Objective: Given a set of determinants, he will evaluate them by triangular form method, at home (75%).

GOAL: The student will retain knowledge gained in the study of chapter 7.

Objective: At the end of the chapter, he will answer questions on a quiz (closed book) made up of problems similar to the REVIEW and Miscellaneous Problems in the chapter (80%).

VIII.

COMBINATIONS AND PERMUTATIONS

GOAL: The student will understand the Fundamental Principle of Enumeration and the concept of factorial.

Objective: He will write solutions to arithmetic problems involving factorial notation, at home (85%).

Objective: He will solve problems such as, "How many four digit numbers are there?" at home (75%).

GOAL: He will understand the concept of a combination.

Objective: The student will calculate various combinations as homework (85%).

Objective: He will solve problems which involve the concept of the tree discussed in example 66-3, as homework (80%).

GOAL: The student will learn the binomial theorem.

Objective: He will prove  $\binom{n}{0} + \binom{n}{2} + \dots + \binom{n}{n} = 2^{n-1}$  as homework (100%).

Objective: Given a set of binomials, he will expand them to given powers, as homework (85%).

Objective: He will prove  $\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \dots + (-1)^n \binom{n}{n} = 0$ , as homework (100%).

GOAL: The student will know how to use the  $\Sigma$ -notations.

Objective: Given a set of terms, he will express it in  $\Sigma$ -notation, as homework (100%).

Objective: Given a  $\Sigma$ -notation, he will expand it to a sum of terms, as homework (100%).

GOAL: The student will retain the knowledge gained in Chapter 8.

Objective: He will write a quiz (closed book) including problems similar to #1-6 of the Chapter Review or #1-3 of Miscellaneous Problems, or the objectives stated previously for this chapter (80%).

## IX

### SEQUENCES

GOAL: The student will know the basic concepts pertaining to sequences.

Objective: Given sequences, he will write specified terms. Homework (85%).

Objective: Given the first few terms of a sequence, he will write the general term. Homework (70%).

Objective: Given a sequence, he will write a given partial sum. Homework (85%).

GOAL: The student will be able to solve problems involving arithmetic progressions.

Objective: Given  $a_1$  and  $d$  (as defined in section 75), he will find  $a_n$  and  $s_n$  ( $n$  given) as homework (90%).

Objective: The student will find sums like  $\sum_{r=1}^n 3r$  and write them as homework (85%).

Objective: He will find the sum of the first  $n$  integers as homework (100%).

GOAL: The student will be able to solve problems involving geometric progressions.

Objective: Given  $a_1$  and  $r$  (as defined in section 76), the student will write  $a_n$  and  $s_n$  for specified  $n$  - homework (90%).

Objectives: He will evaluate sums such as  $\sum_{n=1}^{10} 2^n$  as homework (GO1).

Objectives: The student will prove  $\sqrt[n+1]{r} = \sqrt[n]{r^n} \cdot \sqrt[n]{r}$  as homework (GO2).

GOAL: The student will understand the concept of n-th geometric and arithmetic mean.

Objectives: Given a list of pairs of numbers, he will find n-th geometric and arithmetic means (n given) as homework (GO3).

GOAL: The student will return knowledge learned in this chapter.

Objectives: On a quiz (closed book), he will answer questions similar to those of the review 1-7 or to any from the objectives of the chapter (GO4).

## INVERSE FUNCTIONS AND TRIGONOMETRIC EQUATIONS

GOALS: The student will know what an inverse function is and how it is related to a function.

Objectives: Given a list of functions, he will write whether or not they have inverses and if they do, he will graph both the function and its inverse on the same set of axes, at home (85%).

GOALS: The student will understand arcsine and arccosine functions.

Objective: In class, he will define arcsine and arccosine and write the changes made in sine and cosine for these functions to exist (100%).

Objective: He will solve equations involving trig functions and arcsine, arccosine, as homework (30%).

GOALS: The student will understand the arctan and arccot functions.

Objective: In class, the student will define arctan, arccot, stating how one must modify tan and cot to obtain inverses (100%).

Objective: He will prove a set of trig identities, as homework (30%).

Objective: He will evaluate trig functions involving inverses as homework (30%).

Objective: He will graph trig functions involving inverses, as homework (85%).

Skills: The student will be able to solve equations involving trig functions.

Objective: Given a list of equations, he will write all roots, as homework (8%).

Skills: The student will retain the knowledge gained from the study of chapter 10.

Objective: The student will answer questions similar to those found in the review and miscellaneous problems (85%).

41.

\* ANALYTIC GEOMETRY

GOAL: The student will understand the various representations and characteristics of line.

Objective: Given the equations of lines, he will find their slopes and Y-intercepts, at home (90%).

Objective: Given the equations of lines he will write them in two alternative forms, at home (85%).

Objective: The student will answer p. 371, #4 - 8, at home (75%).

GOAL: The student will understand the circle from an analytic point of view.

Objective: Given the center points and radii of circles, he will find the equations of the circles, at home (80%).

Objective: Given the equations of circles, he will find their radii and center points, at home (80%).

Objective: He will answer p. 374, #3,4,6,8,9 (80%).

GOAL: The student will be able to translate axes and will understand symmetry.

Objective: Given equations, he will state whether they are symmetric about a line, a point, or nothing, and state the line or point, if it exists, at home (80%).

Objective: Given two coordinate systems and points whose coordinates are in the first system, the student will write their coordinates in the second system, homework (20%).

Objective: Given equations and a new coordinate system, he will convert the equations to the new system, homework (75%).

Objective: He will answer #. 370-9, 13, 6, 8, 9, 10 at home (10%).

GOAL: The student will understand the concept of an ellipse.

Objective: He will label all parts on a picture of an ellipse in a quiz (90%).

Objective: Given the equations of ellipses, he will graph them, at home (100%).

Objective: Given the center and length of diameters, and direction of major diameter, he will graph the ellipses, at home (100%).

Objective: Given the equations of ellipses, he will write the foci and major and minor diameter lengths, at home (75%).

GOAL: The student will understand the concept of hyperbola.

Objective: He will label all parts on a picture of a hyperbola in a quiz (90%).

Objective: Given the equations of hyperbolas, he will graph them, at home (100%).

Objective: Given the asymptotes, he will graph the hyperbolas, at home (100%).

Objective: Given the equations of hyperbolas, he will write the foci and asymptotes, at home (75%).

GOAL: The student will understand the concept of the parabola.

Objective: He will label all parts on a picture of a parabola in a quiz (90%).

Objective: Given **equations** of parabolas, he will graph them, at home (100%).

Objective: Given the focus and directrix, he will graph parabolas, at home (100%).

Objective: Given the equations of parabolas, he will write the focus and directrix, at home (75%).

GOAL: The student will understand the class of functions called conics.

Objective: He will define the terms related to conics as a quiz (100%).

Objective: Given conics, he will write eccentricities and directrices, as homework (80%).

Objective: Given focus, directrix, eccentricity, he will write the equation of and graph the conic as homework (80%).

10.2: Given a point in 3D space, find its coordinates.

objectives: Given polar coordinates, he will write the corresponding rectangular coordinates, to work (75%).

objectives: Given equations in polar form, he will convert them to rectangular coordinates, at home (75%).

objectives: Given equations in polar form, he will graph them, to work (75%).

10.3: The students will be able to interpret standard equations into polar form.

objectives: He will sketch graphs of given conics, at home (50%).

objectives: He will derive the formulas for parabola, ellipse, ellipse, parabola, in polar form -s to work (75%).

10.4: The student will be able to relate  $x^2 + y^2 = r^2$ .

objectives: In 20 words or less, he will write what rotation of axes is useful. In class (100%).

objectives: Given one angle of rotation, he will find the new coordinates of the point, at home (8%).

objectives: Given the equation  $x^2 + y^2 + 2xy + 2x + 4y + 1 = 0$ , he will derive the angle through which axes must be rotated to eliminate the cross-product term, at home (10%).

Skills: The student will retain the knowledge gained in the study of chapter 14.

Objective: In a quiz (closed book), he will answer questions such as those found in the review problems or on an objective or in chapter 14.

COLLEGE ALGEBRA OBJECTIVES: SET # 3

## VI. OBJECTIVES FOR GRADE 7, GRADE 8, AND GRADE 9.

### WEEK 1: THE NUMBER SYSTEM

#### A. MAJOR CONCEPTS

1. The basic laws of the number system. (Note: there are no frightening anomalies, very powerful, but they are also very simple and usually easily intuitively obvious.)
2. The real line, including negative numbers. (Graphing in one dimension.)
3. Addition, subtraction, multiplication, and division of real numbers on the number line.
4. Mathematical conventions of order of operations, including the use of parentheses in algebraic expressions.

#### B. OBJECTIVES (INCLUDING SAMPLE TEST ITEMS)

1. Out of class, the student will write a paper of not more than  $\frac{1}{2}$  page justifying the following three steps of the solution of  $2x + 5 = 11 + x$ .

$$\begin{aligned}2x + 5 &= 11 + x \\3x + 5 &= 11 \\3x &= 6 \\x &= 2\end{aligned}$$

2. In an hour exam, the student will match the names of the 15 basic laws (on pages 3-6 of the text #1-10 and 14-17) with their algebraic statements with 90% accuracy.
3. In an hour exam, the student will choose the true algebraic statements from a list containing both true and false statements.

Sample list: (a, b, c, and d represent any real numbers)

1. If  $a = b$ , then  $b = a$
2. If  $c = b$  and  $c = d$ , then  $a + c = b + d$
3. If  $a = b$  and  $c = d$ , then  $a + b = c + d$

Unit 3 : Rational Numbers

A. Major Objective

1. The student will be able to:
  - application of the rules of arithmetic specifically to the rational numbers.

B. Objective 3 (Extending Unit 3: 10-13)

1. Out of class, the student will write a paper of not more than 1 page justifying the following 10 steps of the reduction of  $1\frac{1}{2}$  and  $7/10$ .

$$\left(\frac{1}{2} + \frac{7}{10}\right) = \frac{2}{2} + \frac{7}{2} = \frac{9}{2} = \left(9 \cdot \frac{1}{2}\right) + \left(7 \cdot \frac{1}{2}\right) = \frac{9}{2}$$

$$\left(9 \cdot \frac{1}{2}\right) = \frac{9}{2} = \frac{9}{2} \cdot \frac{1}{10} = \frac{9}{20} = \frac{9}{20}$$

$$\left(7 \cdot \frac{1}{2}\right) = \frac{7}{2} = \frac{7}{2} \cdot \frac{1}{10} = \frac{7}{20} = \frac{7}{20}$$

2. Out of class, the student will write a paper of not more than 1 page presenting a step-by-step solution and justification of  $\frac{3}{4} \cdot 2\frac{1}{3}$ . (Answer =  $1\frac{3}{4}$ )

3. Out of class, the student will write a paper of 1/4 to 1/2 page simplifying

$$\frac{12x^2y^3}{9st^2} \cdot \frac{9t}{12st}$$

step-by-step, justifying each step. (Answer =  $\frac{3}{st}$ ).

4. In an hour exam, the student will multiply or divide 5 pairs of rational numbers and simplify the answers, with 80% accuracy.

Sample pairs:  $\frac{3x}{ab^2} \cdot \frac{3y}{y-x}$ ;  $\frac{3h}{b^2} \cdot \frac{2}{h^2-9} + \frac{2}{h-3}$

5. In an hour exam, the student will add or subtract 5 pairs of rational numbers and simplify the answers, with 80% accuracy.

Sample pairs:  $\frac{a+b}{a^2} + \frac{b}{ab^2}$ ;  $\frac{(x+2)(x-1)}{x^2-y^2} - \frac{2}{2x+2y}$

## UNIT 2 : EXPANSION

### A. MAJOR GOALS (3)

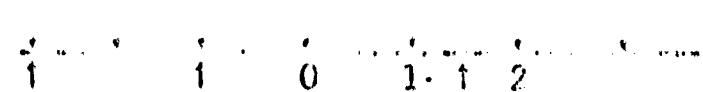
1. Expansion of the following type:
  - 1a. Difference of perfect squares (including completing the square)
  - 1b. Factoring the general quadratic trinomial

### B. OBJECTIVES (ITEMS STUDENT MUST KNOW)

1. Out of class, the student will write a page of not more than 1 page expanding  $(a+4)(2a-1)$ , justifying each step. (Final expansion should be  $2a^2 + 7a - 4$ ). Also suggest an abbreviated procedure which is equivalent to your step-by-step expansion.
2. In an hour exam, the student will expand 10 algebraic expressions with 80% accuracy.  
Sample expressions:  $(a+3)(a-3)$  ;  $(a+b)(a+b)$
3. In an hour exam, the student will factor 10 algebraic expressions with 80% accuracy.  
Sample expressions:  $(x^2 + 8x + 15)$  ;  $(4x^2 - 25)$

4. In an hour's time, the student will plot the 10 indicated points on the given 11-unit graph with 90% accuracy.

Sample graph:



5. In an hour's time, the student will choose the correct graphical representation for each of 5 procedures with 80% accuracy.

Sample procedure:  $2 \cdot 3 + 4$

a)

b)

c)

d)

6. In a unit quiz, the student will correctly evaluate 5 mathematical expressions containing parentheses and brackets with 80% accuracy.

Sample expression:  $5(a + 1) + (3 \cdot 4)$

## UNIT 4 : EQUATIONS, INEQUALITIES, AND POLYNOMIALS

### A. MATERIALS

1. Recognizing equations using basic terms
2. Basic properties of inequalities
3. Graphs of solution sets
4. Word problems involving simple linear equations

### B. OBJECTIVES (INCLUDING SAMPLE TEST ITEMS)

1. Out of class, the student will write a paper of not more than 1/2 page solving the equation

$$3x + 2 = x + 6$$

for  $x$  and justifying each step. (Answer:  $x = 4$ ).

2. In an hour exam, the student will solve each of 10 equations for the indicated unknown with 90% accuracy.

Sample items: Solve  $\frac{5x+5}{2} = 2x$  for  $x$ ;

$$\text{Solve } y = mx + b \text{ for } m$$

3. In an hour exam, the student will find the solution set for 5 simple inequalities with 60% accuracy.

Sample inequality:  $|x + 3| < 8$

4. In an hour exam, the student will translate 8 word problems into algebraic equations (with defined symbols) and solve the equations for the indicated unknown with 70% accuracy. (Correct translation is worth 4 points; solution is worth 1 point).

Sample word problem: If you make 36 monthly payments of \$65 after putting \$500 down on a \$2500 car, how much total interest do you pay?

## UNIT 6 : FUNCTIONS AND GRAPHS

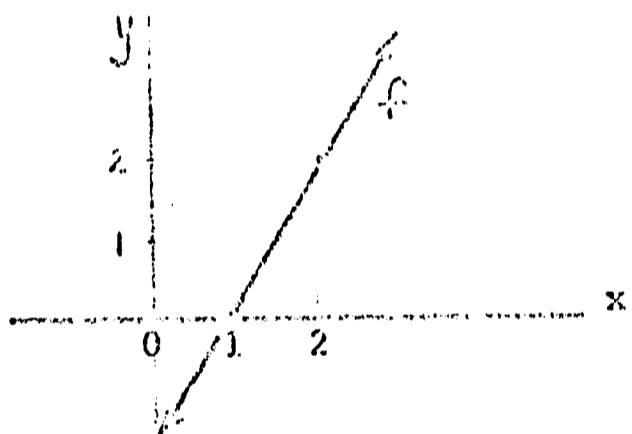
### A. MAJOR CONCEPTS

1. Extension of graphing to two dimensions
2. Notation of functions
3. Straight line graphs
4. Inequality graphs

### B. OBJECTIVES (EXCLUDING SAMPLE TEST ITEMS)

1. Out of class, the student will write a paper (about 1 page) deriving the equation of the line determined by the two points (2,1) and (4,6) using the Pythagorean theorem and the proportional sides theorem of similar (right) triangles from geometry. (Correct equation is  $y = 5/2 x + 4$ ).
2. In an hour exam, the student will choose the function (from 4 choices) which characterizes the given graph (4 of 5 correct).

Sample graph and choices:



- a)  $f(x) = 2x - 2$
- b)  $f(x) = 2x + 1$
- c)  $f(x) = x - 2$
- d)  $f(x) = x + 1$

3. In an hour exam, the student will solve 10 graphic or function problems with 80% accuracy. Each problem will present limited information about a function and ask for other information derivable from the data given.

Sample items:

- a) What is the equation of the straight line with slope 1 which contains the point (2,2)?
- b) What is the slope of the line described by  $2y = 5x - 4$ ?
- c) What is the x-intercept of the line determined by (2,-3) and (4,0)?
- d) Is the point (1,2) in the function determined by  $y \geq x + 3$ ?

## UNIT 5 : EXPONENTS, RADICALS, SCAFFOLDING ROTATION

### A. MAJOR CONCEPTS

1. Laws of exponents (positive, negative, fractional, literal)
2. Scientific notation
3. Operations involving radicals
4. Brief introduction to imaginary numbers

### B. OBJECTIVES (INCLUDING SAMPLE TEST ITEMS)

1. In an hour exam, the student will simplify 10 expressions involving exponents with 80% accuracy.  
Sample expressions:  $8^{\frac{5}{2}}$ ;  $m^a \cdot m^{a+2}$
2. In an hour exam, the student will convert 5 numbers to standard scientific notation, and will reverse the process for 5 numbers given in scientific notation, with 90% accuracy.  
Samples: Convert 10,500,000 and 0.00025 to scientific notation;  
Expand  $6.1 \times 10^{-2}$  and  $10^3$
3. In an hour exam, the student will perform the indicated operation between two radicals in 10 problems with 80% accuracy.  
Sample items:  $\sqrt{28} + \sqrt{70}$ ;  $\sqrt{2} \cdot \sqrt{18}$
4. In an hour exam, the student will express each of 5 radicals in a form involving with 80% accuracy.  
Sample item:  $\sqrt{-50}$

## UNIT 8 : SYSTEMS OF EQUATIONS

### A. MAJOR OBJECTIVES

1. Extension of methods to more than one equation.
2. Graphing of quadratic equations (circles, ellipses, parabolas, and hyperbolas) and simultaneous equations
3. Solving word problems which involve simultaneous equations

### B. OBJECTIVES (INCLUDING SAMPLE TEST ITEMS)

1. Out of class the student will write a paper of about 1 page solving for values of  $x$  and  $y$  which simultaneously satisfy

$$x + 2y = -8 \quad \text{and} \quad 2x + y = -1.$$

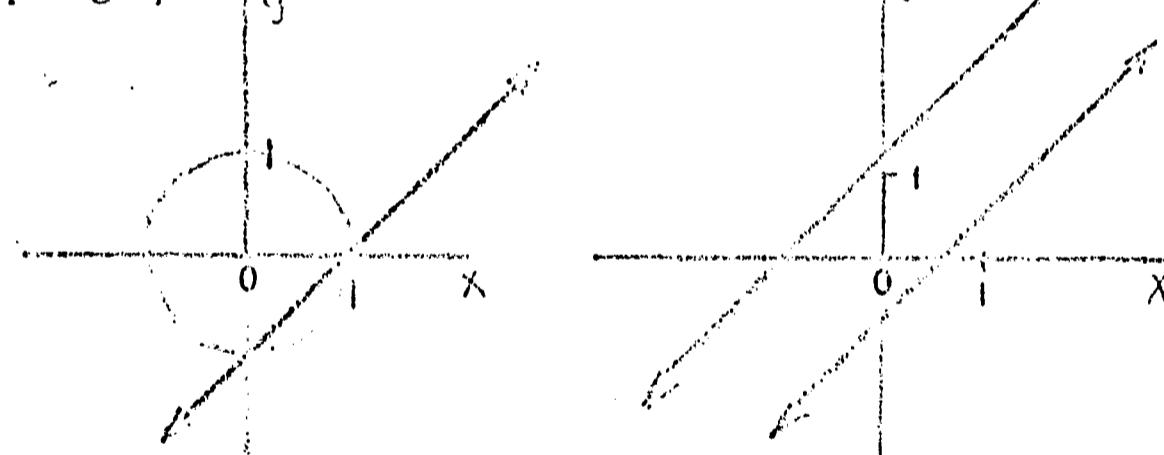
Justify each step by the laws of Chapter 1. (Solution: (-2,3)).

2. On an hour exam, the student will solve 5 pairs of simultaneous equations with 80% accuracy.

Sample item:  $x^2 + y^2 = 2.5$  and  $2x + y = 5$

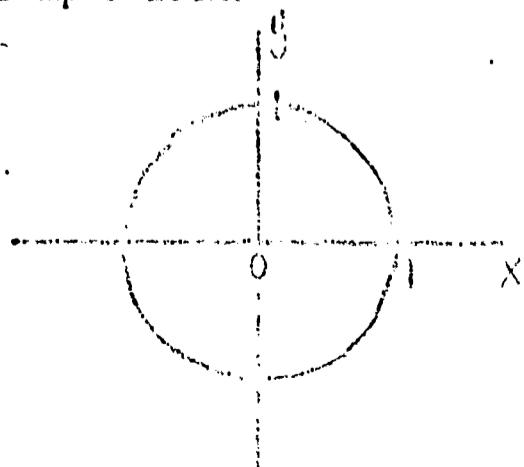
3. On an hour exam, the student will find the solution set of two simultaneous equations from their simultaneous graph with 100% accuracy on 5 sets.

Sample graphs:



4. On an hour exam, the student will choose the function which is graphed, with 80% accuracy on 5 sets.

Sample item:



- a)  $x^2 + y^2 = 1$
- b)  $x^2 + 2y^2 = 1$
- c)  $x + y = 1$
- d)  $x^2 - y^2 = 1$

## UNIT 7 : QUADRATIC EQUATIONS

### A. MAJOR CONCEPTS

1. How to solve equations in which the unknown appears to the second power (factoring, completing the square, quadratic formula).
2. Solving word problems which involve quadratic equations.

### B. OBJECTIVES (INCLUDING SAMPLE TEST ITEMS)

1. Out of class, the student will solve the quadratic equation

$$x^2 + x - 6 = 0$$

by the three methods which we have studied (factoring, completing the square, quadratic equation). Show your methods and solutions in a 1 page paper. (Note: if you don't obtain the same roots with all three methods, you've done something wrong!)

2. In an hour exam, the student will solve 10 quadratic equations (by any method) with 80% accuracy.

Sample equations:  $2x^2 + 7x + 5 = 0$  ;  $x^2 + x = 5$

3. In an hour exam, the student will choose the correct algebraic translation of 5 word problems, with 80% accuracy.

Sample item: The shorter leg of a right triangle is 3 inches less than the longer leg. If the hypotenuse is 15 inches, what is the length,  $x$ , of the shorter leg?

a)  $x^2 + (x + 3)^2 = (15)^2$

b)  $\frac{x(x+3)}{2} = (15)^2$

c)  $[x + (x+3)]^2 = (15)^2$

d)  $\frac{x(x+3)}{2} = 15$

2

## UNIT 9 : RATIOS AND PROPORTIONS

### A. MAJOR CONCEPTS

1. Definitions and uses of ratio and proportion
2. Variation as a function

### B. OBJECTIVES (INCLUDING SAMPLE TEST ITEMS)

1. On an hour exam, the student will compute the required number from the given ratio (with 4 of 5 correct).

Sample item: If the ratio of men to women is 3 to 1, and there are 15 men, how many women are there?

2. On an hour exam, the student will compute the value of  $x$  from the given proportion or variation.

Sample items:  $x$  is to 10 as 3 is to 30 ;

If  $x$  varies as the square of  $y$ , and  $x = 2$  when  $y = 1$ , find the value of  $x$  when  $y = 2$ .

## UNIT 10 : EXPONENTIAL AND LOGARITHMIC FUNCTIONS; APPROXIMATIONS

### A. MAJOR CONCEPTS

1. The basic properties of logarithms and how to apply them to solve problems.
2. The exponential graph
3. Approximations and quick mental calculations and their uses.

### B. OBJECTIVES (INCLUDING SAMPLE TEST ITEMS)

1. Out of class, the student will write a paper of not more than 1 page showing how he found the approximate value of  $(9.8)^{1/4}$  by using 4-place common logarithms. (Note: the Handbook of Chemistry and Physics, available at the library reference desk, has complete log tables and brief instructions for their use.)
2. On an hour exam, the student will convert 5 logarithmic equations to exponential form, and 5 exponential equations into log form, with 70% accuracy.

Sample items: Convert  $2^4 = 16$  to logarithmic form.

Convert  $\log_3 81 = 4$  to exponential form

3. In a separately timed section of the unit quiz, the student will be given 10 algebraic and verbal descriptions of problems, and will choose the answer or closest answer from the four choices offered (90% accuracy in 10 minutes).

Sample items:

$$.015 \times 12 \times \$500 =$$

a) \$10      b) \$50      c) \$90      d) \$140

HONEST SAM'S AUTO REPAIR	
parts	\$16.73
labor	\$27.41
student <u>discount</u>	10%
tax	5%
Total	?

Total is  
closest to:

a) \$39  
b) \$42  
c) \$45  
d) \$49

COLLEGE ALGEBRA OBJECTIVES: SET # 4

#### IV Units of Instruction

##### A. Simple equations, fundamental operations, products and factoring (unit 1)

1. The following main topics will be covered: the formula, the real-number system, algebraic combinations, rules of signs, the equation, solving linear equations in one unknown, solving problems by means of equations, fundamental operations and proofs of the laws of signs, symbols of grouping, the product of two multinomials, zero in multiplication, division of monomials, zero in the division process, division involving polynomials, the product of two binomials with corresponding terms similar, the square of a polynomial, factoring, factors of a quadratic trinomial, trinomials that are perfect squares, factors of a binomial, common factors, factoring by grouping, trinomials reducible to the difference of two squares. The terms set, elements, subset, numerical coefficient, literal number, exponent, monomial, and multinomial, all composing the basic parts of an algebraic expression are introduced. The addition and subtraction of monomials and multinomials is shown. The use of parenthesis as a means of grouping terms is introduced. The multiplication and division of a monomial by a monomial, the multiplication of a multinomial by a monomial, the multiplication of a multinomial by a multinomial, and the division of a multinomial by a monomial is described. Finally, the various forms of factoring are discussed.
2. Given a linear equation in  $x$  the student will be able to solve for the value of the unknown  $x$ . Given a list of algebraic expressions, the student will be able to first select monomials from multinomials and second to perform correctly (written examination) the operations of addition, subtraction, multiplication and division of monomials with monomials, multinomials with monomials, or multinomials with multinomials, and finally factor the algebraic expressions to their least common form.

##### B. Fractions and fractional equations (unit 2)

1. The terms numerator, denominator, member, reciprocal, and lowest common multiple (L.C.M.) are introduced. First, fractions will be converted to lowest terms. Then, the multiplication and division of fractions is introduced. Next, the concept of the L.C.M. is introduced and is applied to the addition and subtraction of fractions. Then we use the previous principles to operate on complex fractions. Finally, we show how the L.C.M. is used to solve fractional equations of the first degree.
2. Given several fractions, the student will be able to perform correctly (written examination) the operations of multiplication, division, addition and subtraction of these fractions and convert the result to lowest terms. Given a complex function, the student will be able to (written examination) correctly simplify the fraction. Given a fractional equation of the first degree, the student will be able to solve (written examination) for the unknown.

### C. Functions and graphs (unit 3)

1. The unit is broken up into the following topics and concepts: constants and variables, functions and functional notation, functions of more than one variable, the rectangular coordinate system, the graph of a function, linear functions.
2. Given a function of one or more variables, the student will be able to graph the function on graph paper.

### D. Linear equations (unit 4)

1. The following topics and concepts compose the unit: independent, inconsistent, and dependent equations, elimination by addition or subtraction, elimination by substitution, three linear equations in three unknowns, solution of two linear equations in two unknowns by means of determinants, determinants of order 3, solution of a system of three linear equations.
2. Given a system of linear equations in two or three variables the student will be able to state correctly (written examination) whether the equations are consistent or inconsistent and if they are consistent find their intersection points. Given a system of non-linear equations the student will be able to use (written exam) the method of substitution to find their intersection points.

### E. Exponents and radicals (unit 5)

1. The unit is divided into the following topics: laws of positive integral exponents, negative integral exponents, fractional exponents, laws of radicals, applications of  $k\sqrt{ab} = k\sqrt{a} k\sqrt{b}$ , applications of  $k\sqrt{\frac{a}{b}} = \frac{k\sqrt{a}}{k\sqrt{b}}$ , changing the order of a radical, rationalizing binomial denominators, addition of radicals.
2. Given radicals of various forms, the student will be able to simplify them, add them, multiply them, and change their order (written examination).

### F. Quadratic equations in one and two variables (unit 6)

1. The unit is divided into the following topics: imaginary numbers, solving a quadratic equation by factoring, solving a quadratic equation by completing the square, complex numbers, the quadratic formula, nature of the roots of a quadratic equation, sum and product of the roots, radical equations, equations in quadratic form, graphs of quadratic equations in two unknowns, graphical solution of two quadratic equations in two unknowns, algebraic solutions, elimination by substitution, elimination by addition or subtraction, elimination by a combination of addition or subtraction and substitution, symmetric equations.
2. Given quadratic equations in one or two unknowns, the student will be able to (written examination) solve for their roots.

G. Ratio, proportion, and variation. Logarithms. (unit7)

1. The terms extremes, means, alternation, inversion, direct variation, inverse variation, joint variation, constant of variation, characteristic mantissa are introduced.
2. Given functions that vary proportionately to one another the student will be able to write the correct equation. Given a number, the student will be able to compute its logarithm.

COLLEGE ALGEBRA OBJECTIVES: SET # 5

A. Class assignments and lectures shall be based upon the textbook for the course, College Algebra, by Beckenbach, Drooyan, and Wooten.

## UNIT I      OVERVIEW AND ORGANIZATION OF COURSE

### I. Major goals of course.

A. To prepare the students for more advanced mathematics in which the algebra will be employed as a tool.

B. To provide the student with an understanding of the structure of mathematics.

C. To provide the student with the background in logic necessary to be able to use the power of mathematics to solve problems.

### II. Specific goals of course.

A. The student will demonstrate his understanding of the structure of the algebra by:

1. Preparing, under test conditions in the classroom, a graph of the structure of the set ( $\mathbb{R}$ ) of real numbers, listing the Binary operations applicable and the Postulates applicable to each structural level.

2. Citing examples of each structure.

3. Solving problems requiring the knowledge of these properties.

B. The student will demonstrate his readiness to move on to more advanced mathematics by completing a final examination on the several topics in this course with a score of 80% or higher under the usual classroom test conditions.

C. The student will demonstrate his understanding of

the logic of the algebra by applying the structure to a set (R) of real numbers to sets of imaginary (I) and complex (C) numbers in the solution of problems on an examination in the classroom under the usual classroom conditions. A minimum score of 70% shall be considered as attainment of this objective.

## UNIT II PROPERTIES OF REAL NUMBERS

### I Statement of major concepts, goals.

- A. The student will learn the terms basic to the discussion of the properties of real numbers.
- B. The student will understand the theorems defining real numbers and field properties.
- C. The student will be able to apply these concepts to problems involving real numbers.

### II. Specific objectives.

- A. The student shall define the terms presented in this unit. This shall be done by 1) defining, and 2) giving an example of each term, in a closed book examination at the end of the unit.
- B. The student shall demonstrate his understanding of operations on sets by solving selected problems on a closed book examination at the end of the unit.
- C. The student shall demonstrate his understanding of the classification of numbers by defining and giving examples of natural, rational, irrational, real, imaginary, and complex numbers on a closed book examination at the end of the unit.
- D. The student shall demonstrate his understanding of the Field Postulates and axioms by using them in an open book examination to prove that a set  $F$  (of real numbers) is a field  $R$  of real numbers. The student shall also apply these postulates and axioms to problems.

E. The student shall demonstrate his understanding of theorems concerning field properties by applying them to selected problems in a closed book examination.

F. The student shall demonstrate his understanding of order and completeness by applying these concepts to the solution of selected problems in a closed book examination.

## UNIT III POLYNOMIALS

### I. Major concepts-goals

- A. To learn the terms basic to the discussion of the properties of polynomials.
- B. To apply the axioms and postulates to a polynomial  $P$  over the field  $R$  of real numbers in order to prove that  $P(x) \in R [x]$
- C. To understand and apply the properties associated with products and quotients of polynomials.
- D. To understand and apply the properties associated with open sentences in the solution of inequalities.

### II. Specific objectives

- A. The student shall demonstrate his understanding of the terms presented in this unit by: 1) defining them and 2) giving an example of each of them, on a written examination (closed book) at the completion of the unit.
- B. The student shall use the list of axioms, postulates, and laws to prove that if  $P(x)$  is a polynomial over the field  $R$  of real numbers then  $P(x) \in R [x]$  and  $P(x)$  is an integral domain.
- C. The student shall demonstrate his understanding of the principles of synthetic division by: 1) proving that if  $P(x) = (x-c) Q(x) + r$  then  $\frac{P(x)}{x-c} = Q(x) + \frac{r}{x-c}$   $x \neq c$   
2) solving typical problems using this principle, on a closed book written examination at the end of

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the unit.

D. The student shall demonstrate his understanding of the principles of solution of inequalities by solving typical problems of this type on a written closed-book examination at the end of the unit.

#### UNIT IV RELATIONS AND FUNCTIONS

##### I. Major concepts-goals

- A. To learn the terms and definitions basic to the discussion of relations and functions.
- B. To apply the principles of set theory in the solution of problems involving relations and functions.
- C. To understand theory concerning linear and quadratic functions and to apply theory to the solution of problems.
- D. To graph inequalities and apply set theory to the solution of special problems.

##### II. Specific objectives.

- A. The student shall demonstrate his understandings of the terms and definitions concerning relations and functions by 1) defining them and 2) giving an example of each of them on a written classroom examination at the conclusion of this unit.
- B. The student shall use the axioms and definitions to solve problems concerning cartesian products, open sentences in two variables, relations, linear and quadratic functions, and graphs of linear and quadratic functions.

## UNIT V EXPONENTIAL AND LOGARITHMIC FUNCTIONS

### 1. Major concepts-goals.

- A. To prove that powers with irrational exponents are real numbers.
- B. To use the theorems concerning exponential and logarithmic functions in the solution of problems.

### II Specific objectives.

- A. The student shall demonstrate his understanding of the properties of an exponential function by solving problems involving the theorems defining these properties in a written examination at the conclusion of this unit.
- B. The student shall demonstrate his understanding of the theorems concerning logarithmic functions by solving selected problems requiring their application on a written examination at the conclusion of the unit.

## UNIT VI MATRICES AND DETERMINANTS

### I Major concepts-goals

- A. To use matrices and determinants in the solution of problems.
- B. To prove: If A and B are  $N \times N$  matrices with real number entries, the system is a ring with an identity.
- C. To understand the basic differences between group properties of numbers and the field and integral domain properties and their relationship to other sets of numbers.
- D. To understand the application of matrix multiplication theorems to problems.
- E. To learn the properties of determinants and their application to the solution of problems.
- F. To understand the application of Cramer's rule to determinants.

### II Specific objectives.

- A. On a written examination at the conclusion of the unit the student shall:
  1. Demonstrate his understanding of matrix addition and multiplication by solving problems requiring the use of these properties.
  2. Prove that the set  $S_{nxn}$  of  $nxn$  Matrices is a ring with an identity. Theorems will be provided for student use.
  3. Demonstrate his understanding of theorems

applying to determinants by solving problems requiring their application.

4. Demonstrate his understanding of Cramer's Rule by using it to solve specific problems.

1. Major concepts-goals

- A. To understand the use of establishing properties of real numbers in the consideration of other sets of numbers.
- B. To investigate the properties of the set C of complex numbers.
- C. To understand the application of theorems concerning complex numbers to quadratic equations and vectors.

II Specific objectives,

- A. The student shall demonstrate his understanding of the theorems concerning complex numbers by proving that the set C of complex numbers is a field.
- B. The student shall demonstrate his understanding of the use of ordered pairs by solving problems requiring their use.
- C. The student shall demonstrate his understanding of the application of complex numbers to quadratic equations by solving problems involving these concepts.
- D. The student shall demonstrate his understanding of the use of ordered pairs as vectors by solving problems involving these concepts.

## UNIT VIII      THEORY OF EQUATIONS

### I. Major concepts-goals

- A. To investigate the properties of the polynomial equation  $P(x) = A_0x^n + A_1x^{n-1} + \dots + A_n$ ,  $A \neq 0$ ,  $a_i \in C$  or  $a_i \in R$  over their respective fields.
- B. To understand the application of the synthetic division theorem to complex numbers.
- C. To understand the application of real, rational, irrational and complex zeros of polynomial functions.
- D. To understand Descartes' Rule of signs.

### II Specific objectives.

- A. The student shall demonstrate his knowledge of the application of the real number theorems to the set of complex numbers by solving selected problems on a written examination.
- B. The student shall demonstrate his knowledge of the fundamental theorem of algebra by demonstrating its proof and application to selected problems on a written examination.
- C. The student shall demonstrate his understanding of the application of Descartes Rule of Signs in the solution of problems by illustrating its application on a written examination.
- D. The student shall demonstrate his understanding of the solution of the general polynomial for real, rational, irrational, and/or complex zeros by solving problems for these zeros on a written exam.

UNIT IX PROBABILITY

I. Major concepts-goals

- A. To understand the basic counting principles, permutations and combinations.
- B. To understand the application of permutations and combinations to sample spaces and events.
- C. To understand the application of the basic counting principles to a function  $P$  on a sample space  $S$ .
- D. To understand the application of theorems of Probability to solution of specific problems.

II. Specific objectives.

- A. The student shall demonstrate his understanding of the basic counting principles of permutations and combinations by 1) defining terms, and 2) by giving illustrations of these terms, on a written examination.
- B. The student shall demonstrate his understanding of the application of theorems of permutations and combinations by solving specific problems on a written examination.
- C. The student shall demonstrate his understanding of the applications of permutation and combination theory to sample spaces and events by solving specific problems in a written examination.
- D. The student shall demonstrate his understanding of independent and dependent events by solving specific problems on a written examination.